The Relationship Between Implied and Realized Volatility of S&P 500 Index

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Abstract
This paper studies the relationship between implied and realized volatility by using daily S&P 500 index option prices over the period between January 1995 and December 1999. In particular, we want to test the how different measurement errors affect the stability of this relationship. Two sources of measurement errors are considered. The first one is the measurement error in realized volatility. Four different estimators of computing realized volatility are tested. They are the standard deviation of daily return; the Parkinson (1980) extreme value volatility estimator, the Yang & Zhang (2000) range estimator, and the square root of intraday return squares (Andersen, 2000). The second source of error comes from model specification. The implied volatility computed from Black-Scholes model is compared with that from calibrated Heston (1993) stochastic volatility option-pricing model. We find the improvement of the measurement of realized volatility can significantly improve the forecast ability of implied volatility, with the realized volatility estimated from intraday return data the most predictable. However, there is no significant difference in forecasting realized volatility using implied volatility either from Black-Scholes model or from Heston model. When both implied volatility and historical volatility are used to forecast realized volatility, we find implied volatility outperforms historical volatility and even subsumes information of historical volatility. This result holds for all measurements of realized volatility and implied volatility.

1. Introduction

Option markets are often considered as markets for trading volatility. It then follows that implied volatility backed out from option price is likely to be a good predictor of subsequent observed volatility if the option market is efficient and the option pricing model is correct. The forecast ability of implied volatility is often compared with the forecast ability of historical volatility calculated from past return information. Since option traders are generally institutional traders and have more information, it is also expected that implied volatility is better in forecasting future volatility than historical volatility. The issue of testing the relationship between the implied volatility and future realized volatility has been the subjects for a number of studies (Canina and Figlewski, 1993; Christensen and Prabhala, 1998; Fleming, 1998, etc). The test between implied and realized volatility also forms a formal test of information efficiency of the option market. Hull and White (1987) show that when volatility is constant, the Black-Scholes implied volatility of an ATM (at-the-money) option approximately equals to the expected future realized volatility during the option life. This means that if option market is information efficient in reflecting underlying asset market, then the implied volatility should form an unbiased forecast of future realized volatility over the option life. However, the regression between implied and realized volatility presents quite mixed results. Canina and Figlewski (1993) find that implied volatility of S&P 100 index option contains no information

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about future volatility. Such a conclusion shakes the base of option pricing theory. Because option price is based on the underlying asset price, it is unreasonable that volatility implied from option market has no relationship with the volatility of the underlying asset unless the index option market is inefficient. However, if one wants to reach this conclusion, he has to eliminate other factors that could possibly distort the relation between implied and realized volatility. For example, Christensen and Prabhala (1998) show that the use of daily overlapping data will significantly overestimate the forecast ability of historical volatility. Instead, they suggest a new sampling at monthly level in order to avoid data overlapping problems. They find that implied volatility, sampled at monthly level, outperforms past volatility in forecasting future volatility and even subsumes the information content of past volatility in some specification. Fleming (1998) tries to correct serial impendence problem by a GMM (Generalized Method of Moments) estimation procedure. Furthermore, he points out that the use of Black-Scholes model to compute American option also leads to a measurement error in implied volatility, so he compute option model price by a modified binominal model developed by Fleming and Whaley (1994) to account for the early exercise problems. He finds that the implied volatility from S&P 100 index options performs better in predicting future realized volatility of S&P 100 returns than historical volatility does, however, the implied volatility is still an upward biased forecast of realized volatility. In another work, Taylor and Xu (1997) compare the forecast ability of implied volatility and historical volatility using Deutschemark/Dollar foreign exchange rate data, they conclude that when realized volatility and historical volatility are constructed from daily data, the implied volatility dominates historical volatility in forecasting realized volatility. However, when intraday 5-minute return is used to construct realized and historical volatility, the historical volatility outperforms implied volatility in forecasting realized volatility. Their results suggest that a better forecast ability could be achieved by improving the measurement accuracy of realized volatility. The existing literatures show that the relation between implied and realized volatility tends to be affected by the measurement of implied volatility, the measurement of realized volatility, the econometric models and even the data set used.

The purpose of this study is to investigate the factors that may affect the relationship between implied volatility and realized volatility. Two sources of measurement errors are considered. The first one is the error in measuring realized volatility. According to this interpretation, the ex-ante volatility forecasts are unbiased and efficient, but the ex-post observed volatility is incorrect, relative to their true distributions, so that it appears that the ex-ante forecast is biased. If this hypothesis is true, then the forecast ability of implied volatility can be improved by constructing more accurate measurement of realized volatility. The optimal way of measuring realized volatility is assessed by comparing the forecast performance of implied volatility with different measurements of realized volatility and historical volatility. More specifically, we test four different measurements of realized volatility, the first one is classical volatility, which is the standard deviation of asset return during the computation period; the second one is the Parkinson extreme value volatility estimator; the third one is the multi-period range estimator newly developed by Yang and Zhang (2000); the fourth one is the integrated volatility used by Anderson (2000). The second error source is in measuring the implied volatility. Previous research (Canina and Figlewski, 1993) attribute the poor forecast ability of implied volatility partly to the misspecified Black-Scholes model. According to them, the option market does contain information in forecasting future realized volatility, however, the Black-Scholes model which assumes the constant volatility is incorrect in reflecting the true volatility process perceived by the market. So the implied volatility backed out from Black-Scholes model cannot reflect market expectation of future volatility. That is why implied volatility virtually has no relationship with realized volatility. Since the main bias associated with the Black-Scholes model is its constant volatility assumption, it is natural to test whether implied volatility computed from more sophisticated stochastic volatility models can improve the forecast ability of the implied volatility. This is done by comparing the forecast ability of implied volatility implied either from Black-Scholes model or from the Heston stochastic volatility model.

The remaining of the paper is organized as follows. Section 2 describes the data and the sampling procedure. Section 3 introduces four measurements of realized volatility. Section 4 deals with computing implied volatility either from Black-Scholes model or from Heston stochastic volatility model. Section 5 examines the relationship between implied volatility and realized volatility using non-overlapping data. Section 6 concludes our finding.

2. Data and sampling procedure

2.1 Data

The option data used in this study are daily close prices of S&P 500 index options. Currently the S&P 500 index options have the largest open interest and the largest trading volume in all options traded on the Chicago Board Options Exchange (CBOE). Unlike S&P 100 index options, S&P 500 index options are European style and do not have the problem of early exercise. Our sample period is from January 1, 1995 to December 9, 1999. The S&P 500 index options expire on the third Friday of each contract month. On each trading day, we report the close price for each strike price and time-to-maturity. The time-to-maturity is measured as the number of calendar days from the trading date to the Thursday immediately preceding the Friday when the option expires, because S&P 500 index options expire at the opening of trading. The reported index level is the closing price of S&P 500 index.

The S&P 500 index option is European style and must adjust for dividends. We use the method suggested by Bakshi, Cao and Chen (1997) to adjust index levels by subtracting the present value of the future dividends during the option life period. U.S Treasury bill yields are used as the proxy of risk-free interest rate. The data is collected from Bloomberg.

The interest rate does not change too much on daily basis; besides, as Rubinstein (1985) points out, the error in interest rate measurement has little effect on the option price. For simplicity, we only use one month T-bill yield to represent the short-term interest rate.

2.2 Sampling procedure

One practical problem in testing the relationship between realized volatility and implied volatility or historical volatility is the optimal selection of sampling frequency. Previous studies show that the different sampling procedure leads to complete different results even for the same data set. Canina and Figlewski (1993) use overlapping daily time series to estimate historical volatility. This introduces a serious autocorrelation problem in the realized volatility because the computed realized volatilities on two consecutive days share all but one-day information. It is not surprising that they find historical volatility outperforms implied volatility in forecasting realized volatility. Because their historical volatility measurement contains part of the information in the realized volatility, thus are highly correlated with the realized volatility. Christensen and Prabhala (1998) point out that the overlapping problem is the major reason that causes the regression in Canina and Figlewski's work inefficient. On the contrary, they suggest a sampling at monthly level to avoid overlapping problem. They find that when sampling at monthly level, the implied volatility dominates historical volatility in forecasting realized volatility. Their results show that using daily overlapping data tends to overstate the explanatory power of historical volatility. In order to avoid overlapping problem, we follow the sampling procedure of Christensen & Prabhala (1998). We only choose the option observed on the Wednesday immediately following the option contract expiration date. The Wednesday data is used because Wednesday has the least holidays. On each Wednesday, we locate the nearest ATM option that expires in the next calendar month. In general, this option has approximately three and a half to four and a half weeks time to maturity. The implied volatility computed from this option price is used to forecast the realized volatility during the same period. Such method avoids the overlapping data. By this way, we sample at lower monthly frequency. Our sample covers a period of five years and has approximately 59 observations. If the Wednesday data is not available, we use the Thursday that immediately follows the Wednesday.

3. Measurement of realized volatility

Our first purpose is to examine how the measurement of realized volatility affects the relationship between implied volatility and realized volatility. The hypothesis is that the forecast ability of implied volatility can be improved by constructing better measurement of realized volatility. The realized volatility is estimated using S&P 500 index return data during the life of the option period. Four different volatility estimators are tested. The first one is the traditional close-to-close volatility estimator that is standard and widely used by the literature as a proxy of realized volatility. The second one is the Parkinson (1980) extreme value estimator that uses daily high and low prices. Unlike the close-to-close volatility estimator, Parkinson estimator does not require the volatility to be constant over the estimation period. We believe that the release of constant volatility restriction will lead to more accurate measurement of realized volatility. The third one is the Yang and Zhang (2000) volatility estimator, which is another range-based volatility estimator that incorporates daily high and low as well as daily open and close prices. This volatility estimator explicitly incorporates the opening jump into pricing formula, thus releases the continuous trading assumptions made by other volatility estimators. Since opening jump always happens when unexpected information comes during non-trading time, the incorporation of opening jump may lead to a better measurement of the realized volatility. The fourth volatility estimator is computed from intraday 5-minute return data. This volatility estimator, named as "integrate volatility" by Anderson (2000), is shown to improve the volatility estimation significantly. Because the high frequency data is closer to the continuous data generating process and contains more information in forecasting future volatility, it is natural to hypothesize that the volatility estimator constructed from high frequency data can improve volatility forecast ability.

We also want to compare the performance of implied volatility and historical volatility in forecasting future realized volatility. It is generally believed that option market is a market of trading volatility and contains more information in forecasting volatility. It is hypothesized that the implied volatility computed from option price should outperform the historical volatility computed from underlying asset return data in forecasting future realized volatility. The historical volatility is computed in the same way as the realized volatility using S&P 500 index return data for the past one month. Because the length of the option maturity is less than one month, it is better just using past one-month return to compute the historical volatility. The mean return is assumed to be zero since our previous study shows that the average daily return of S&P 500 index is not significantly different from zero. In order to be consistent with annualized implied volatility, all realized and historical volatilities are converted to annualized term. The four volatility estimators are defined as follows:

The volatility estimated from daily closing price is

$$Vol_{rel1} = \sqrt{\frac{252}{n}\sum_{t=1}^{n} r_t^2}, \tag{1}$$

where r_t is the daily return on date t, computed as the logarithm of the ratio of two consecutive closing prices. The Parkinson estimator is

$$Vol_{rel2} = \sqrt{\frac{252}{n}\sum_{t=1}^{n}\frac{1}{4\ln 2}(\ln H_t - \ln L_t)^2}, \tag{2}$$

where H_t and L_t are the daily high and low prices on date t. And the Yang and Zhang estimator is

$$Vol_{rel3} = \sqrt{252} \times \sqrt{V_o + kV_c + (1-k)V_{RS}}, \tag{3}$$

where V_o, V_c and V_{RS}, are defined as follows:

$$V_o = \frac{1}{n-1}\sum_{t=1}^{n}(o_t - \bar{o})^2, \quad o_t = \ln O_t - \ln O_{t-1}, \quad \bar{o} = \frac{1}{n}\sum_{t=1}^{n} o_t,$$

$$V_c = \frac{1}{n-1}\sum_{t=1}^{n}(c_t - \bar{c})^2, \quad c_t = \ln C_t - \ln C_{t-1}, \quad \bar{c} = \frac{1}{n}\sum_{t=1}^{n} c_t,$$

$$V_{RS} = \frac{1}{n}\sum_{t=1}^{n}[(\ln H_t - \ln O_t)(\ln H_t - \ln C_t) + (\ln L_t - \ln O_t)(\ln L_t - \ln C_t)],$$

where O_t and C_t are the daily open and close prices on date t, and the constant k, chosen to minimize the variance of the estimator, is given by:

$$k = \frac{0.34}{1.34 + \frac{n+1}{n-1}},$$

Our last volatility estimator is the volatility estimated from 5-minute return series. Since the high frequency data is believed to contain more information in forecasting future volatility. Let $\Delta = T/N$ be a fixed interval of trading time (measured in years) and $R_{t+i\Delta}$ be the return during the period from $t + (i-1)\Delta$ to $t + i\Delta$, the annualized realized volatility over the period from t to $t + T$ is defined as:

$$Vol_{rel4} = \sqrt{\frac{1}{\Delta}\left[\frac{1}{N}\sum_{i=1}^{N} R^2_{t+i\Delta}\right]} \tag{4}$$

In this study, Δ stands for the 5-minute time interval. There are 77 5-minute returns on each trading day.

4. Measurement of implied volatility

Our second concern is whether model specification affects the relation between implied volatility and realized volatility. Previous study argues that the Black-Scholes model mis-prices the option, thus the implied volatility computed from Black-Scholes model does not reflect the true expectation of future realized volatility. Therefore a better forecast ability of implied volatility can be achieved by improving the measurement of implied volatility. The Heston stochastic volatility model is a good candidate because this model allows volatility to follow random process and significantly improves option pricing model. In this study, we directly compare the forecast ability of the implied volatility computed either from Black-Scholes model or from Heston stochastic volatility model.

4.1 Black-Scholes implied volatility

Assuming that asset price, S_t, follows a lognormal process with a constant volatility, σ, the current price of a European call option is given by the Black-Scholes formula

$$\begin{aligned} C_0 &= S_0 N(d_1) - Ke^{-rT}N(d_2), \\ N(d) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d} e^{-x^2/2}dx, \\ d_1 &= \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}, \end{aligned} \tag{5}$$

where S_0 is the current dividend-exclusive index level, K is the exercise price, r is the risk-free interest rate, and T is the time to maturity. For each option, we know the time-to-maturity, exercise price, annualized interest rate, and adjusted index level. The only unobservable parameter in the Black-Scholes formula is the volatility. One may compute the implied volatility that equates the model price with the observed market price.

4.2 Heston model implied volatility

The Heston stochastic volatility option-pricing model is widely tested because of its three main features: it does not allow negative volatility, it allows the correlation between asset returns and volatility and it has a closed-form pricing formula. The model for underlying asset price is

$$\begin{aligned} dS_t &= \mu S_t dt + \sqrt{v_t} S_t dB_{1t}, \\ dv_t &= \kappa(\theta - v_t)dt + \eta\sqrt{v_t}dB_{2t}, \end{aligned} \tag{6}$$

where κ is the mean-reverting speed, θ is the long-run average variance and η is the volatility of volatility, dB_{1t} and dB_{2t} are two Brownian motions with correlation coefficient, ρ. The market price of volatility risk is assumed to be proportional to instantaneous variance level, i.e.,

$$\lambda(S_t, v_t, t) = \lambda v_t,$$

where λ is constant. The Heston model directly transforms the real probability to the risk-neutral probability by explicitly incorporating the risk premium λ into the option pricing model.

To compute option price from the Heston model, one needs the input parameters that are not observable from market data. The input parameters fall into two group, the structure parameters that govern the diffusion process of the underlying asset (μ, κ, θ, η, and ρ), and the spot variance v_t and risk premium λ. Because volatility is random and risk premium λ is unobservable, the exact maximum likelihood function for the Heston stochastic volatility model cannot be traced, so the traditional maximum likelihood estimation cannot be applied to estimate Heston model directly. Of course, on can always use option panel data to back out structure parameters, as Bakshi, Cao and Chen (1997) and Nandi (1998) do. However, the option is priced under risk neutral probability, it is not clear whether the option implied structure parameters truly reflect the original information contained in the underlying asset return distribution. In this study, we adopt a two-step procedure to estimate the stochastic diffusion process as well as the option risk premium jointly. In the first step, the simulation-based indirect inference method is used to estimate the structure parameters that govern the underlying asset process (μ, κ, θ, η). In the second step, the remaining parameters (v_t, λ, ρ) are estimated by a non-linear least square method. The indirect inference method is a simulation based, moment matching procedure. The method works in the following way: suppose the true data generating process is governed by a stochastic diffusion process, one can simulate discrete time observations from this process by the Euler approximation given any set of structural parameters. The simulated data is estimated using some discrete time model called auxiliary model. The market data is also estimated by the same auxiliary model. If the moments from the simulated data match the moments from the market data, which means that the simulated data has the same property as the market data. Then the structural parameters that generate the simulated data represent the true data generating process. In our study, we estimate both the market data and simulated data with the GARCH (1,1) model and search for the structural parameters that match the GARCH parameters of the simulated data to the market data. This estimation yields the following set of parameters, $\kappa = 2.75$, $\theta = 0.035$, and $\eta = 0.425$.

After κ, θ, η are estimated, we need to estimate the correlation coefficient ρ, the spot variance v_t, and the risk premium λ. This is done by a non-linear least square method. On each day, there are many option quotes with different times to maturity and different exercise prices. Some former studies only use at-the-money options, but we use all options available on the particular day to estimate required parameters on that day since more information is included. Define:

$$\varepsilon_t = p - p(\rho, \lambda, v_t) \tag{7}$$

as the error of between the market price and theoretical price computed from Heston model, we want to search for a set of parameters (ρ, λ, v_t) that minimize the sum of square errors:

$$\min_{(\rho,\lambda,v_t)} \sum_{i=1}^{I} \varepsilon_t^2, \tag{8}$$

where I is the total number of options on the day. This is done on each day. Table 1 provides our estimation results.

After we finish the calibration of Heston model, we can compute Heston implied volatility by evaluating the expected average volatility over the remaining life of the option:

TABLE 1: SUMMARY STATISTICS ABOUT THE IMPLIED PARAMETERS (1995-1999)

The reported numbers are long-run average variance, θ, the mean-reverting speed, κ, the variation of volatility, η, the spot variance, v_t, the risk premium, λ, and the correlation coefficient between the asset return and its volatility, ρ. The sample period covers 1,235 days from January 3, 1995 to December 9, 1999.

	θ	κ	η	v_t	λ	ρ
Mean	0.035	2.75	0.425	0.0384	−0.8716	−0.4644
Median				0.0356	−0.5675	−0.5238
Standard Deviation				0.0266	2.4916	0.3147
Minimum				0.0037	−16.1865	−1.8009
Maximum				0.1955	9.8797	1.0488

Source: Original Calculation

$$\begin{aligned} Vol_{implied} &= \sqrt{\frac{1}{T}\int_0^T E_0(v_\tau)d\tau} \\ &= \sqrt{\frac{1}{T}\int_0^T [\theta + (v_0 - \theta)e^{-\kappa\tau}]d\tau} \\ &= \sqrt{\theta + \frac{1 - e^{-\kappa T}}{\kappa T}(v_0 - \theta)}, \end{aligned} \tag{9}$$

where v_0 is the implied instantaneous variance at time 0. The Heston implied volatility defined by equation (9), comparable with Black-Scholes implied volatility, is also used by Poteshman (2000) to study forecasting future volatility from option prices.

5. Regression model

The forecast ability of implied volatility is typically assessed in the literature by the regression between the implied volatility and the future realized volatility:

$$Vol_{realized}(t) = \alpha + \beta Vol_{implied}(t) + \varepsilon(t). \tag{10}$$

If implied volatility contains information in forecasting the future volatility, then β should be significantly different from zero. Moreover, if the implied volatility is an unbiased forecast of realized volatility, then the joint hypothesis that $\alpha = 0$ and $\beta = 1$ can not be rejected.

Table 2 reports the regression result of Black-Scholes implied volatility using different measurements of realized volatility. Three conclusions can be made from the regression results. First, the slope coefficient for implied volatility, β, is significantly different from zero at 99% confidence interval for all realized volatility measurements, indicating that the implied volatility contains information in forecasting future realized volatility. The value of β range from 0.613 for the realized volatility measured by the Parkinson estimator to 0.814 for the realized volatility measured by the close-to-close estimator. Second, the implied volatility is a biased forecast for the future realized volatility. The intercept α is not significantly different from zero. The t-statistics for the null hypothesis that $\beta = 1$ is rejected at 95% confidence interval when realized volatility is measured by the Parkinson estimator, the Yang and Zhang estimator and the volatility constructed by 5-minute return. However, the null hypothesis cannot be rejected when volatility is measured by the close-to-close estimator. But the joint hypothesis that $\alpha = 0$ and $\beta = 1$ is rejected for all measurements of realized volatility at 99% confidence interval. Third, The forecast ability of implied volatility is improved when a better measurement of realized volatility is used. The adjusted R^2 is 0.4364 when realized volatility is constructed from intraday return, but only 0.3633 when realized volatility is constructed from daily return. The results show that using high frequency data significantly improves the predictability of the asset return volatility. Since a better measurement of volatility should lead to better forecast of the future volatility, the result also confirms that using high frequency data can significantly improves volatility measurement. This is not surprising since high frequency data is closer to the continuous time data generating process and contains more information and should provide a better estimation of the true volatility. Our result is consistent with Andersen (2000), which also finds that volatility becomes more predictable when it is measured by the sum of intraday return. The range volatility estimators also outperform traditional close-to-close volatility estimator in their predictability. More specifically, the R^2 is 0.4238 when realized volatility is measured by the Yang and Zhang estimator and 0.3647 when realized volatility is measured by the Parkinson estimator. The range volatility estimator using daily high and low prices in addition to the daily close prices, thus contains more information in forecasting volatility, in particular, Yang and Zhang volatility estimator explicitly incorporates the opening jump and is closer to the real market condition, so the predictability of Yang and Zhang volatility estimator is better than the Parkinson volatility estimator.

TABLE 2: FORECAST REGRESSION OF BLACK-SCHOLES IMPLIED VOLATILITY WITH DIFFERENT MEASUREMENTS OF REALIZED VOLATILITY

$$Vol_{realized}(t) = \alpha + \beta Vol_{BS\_implied}(t) + \varepsilon(t)$$

OLS estimates with t-statistics for $\alpha = 0$ and $\beta = 0$ (in parentheses). Different measurements of realized volatility are used. Realized 1 is computed from daily return, realized 2 is the standard deviation of Parkinson variance estimator, realized 3 is the standard deviation of Yang and Zhang variance estimator, realized 4 is the standard deviation of the sum-square of 5-minute intraday return. All volatilities are annualized. The realized volatility is computed under the restriction that the mean return is equal to zero. The implied volatility is the Black-Scholes implied volatility from the closest ATM options. The F-statistic tests the joint hypothesis that $\alpha = 0$ and $\beta = 1$. The t-statistic in the last column is for the null hypothesis that $\beta = 1$. There is one observation for each Wednesday in the data period that follows an option expiration data, and the time interval covered by each observation extends from that Wednesday to the next option expiration date.

Realized Volatility	α	β	Adjusted R^2	F statistic	T statistic
Realized 1	−0.0135	0.814	0.3633	25.16	−1.334
	(−0.5332)	(5.838)			
Realized 2	0.0101	0.6131	0.3647	74	−3.695
	(0.5334)	(5.856)			
Realized 3	0.0036	0.7312	0.4238	37	−2.429
	(0.1813)	(6.607)			
Realized 4	−0.0151	0.7502	0.4364	65	−2.25
	(−0.7519)	(6.775)			

Source: Original Calculation

Table 3 reports the regression result of implied volatility to realized volatility when implied volatility is computed from the Heston model price. The result is compatible with that in table 2. The slope coefficient of the implied volatility is significant different from zero and the intercept is not significantly different from zero and the joint hypothesis that $\alpha = 0$ and $\beta = 1$ is rejected at 99% confidence interval for all measurements of realized volatility. The adjusted R^2 shows the same pattern as that in table 2. The R^2 is the highest for intraday volatility estimator, the second highest for Yang and Zhang volatility estimator, followed by the Parkinson volatility estimator and daily volatility estimator has the lowest R^2. However, the R^2 is significantly lower than the R^2 in table 2 for all measurements of realized volatility. Taking the intraday volatility estimator as an example, the R^2 decreases 16% from 0.4364 for Black-Scholes implied volatility to 0.3649 for Heston implied volatility. The F-statistic for the joint hypothesis that $\alpha = 0$ and $\beta = 1$ increase from 65 for Black-Scholes implied volatility to 72 for Heston implied volatility, indicating that the Heston implied volatility is more biased forecast of realized volatility than Black-Scholes implied volatility. The pattern is the same when other measurement of realized volatility is used. The result is a little bit surprising since the stochastic volatility model allows volatility to have more flexible distribution and is believed to be closer to the true data generating process than the constant volatility Black-Scholes model. However, the implied volatility backed out from Heston model has less explanatory power and is more biased than the Black-Scholes implied volatility in forecasting future realized volatility. Recall that although the Heston model allows volatility to follow another diffusion process, it still restricts the asset price process and volatility process to some parametric form. If the market true data generate process deviates from this

TABLE 3: FORECAST REGRESSION OF HESTON MODEL IMPLIED VOLATILITY WITH DIFFERENT MEASUREMENT OF REALIZED VOLATILITY

$$Vol_{realized}(t) = \alpha + \beta Vol_{Heston\_implied}(t) + \varepsilon(t)$$

OLS estimates with t-statistics for α = 0 and β = 0 (in parentheses). Different measurements of realized volatility are used. Realized 1 is computed from daily return, realized 2 is the standard deviation of Parkinson variance estimator, realized 3 is the standard deviation of Yang and Zhang variance estimator, realized 4 is the standard deviation of the sum-square of 5-minute intraday return. All volatilities are annualized. The realized volatility is computed under the restriction that the mean return is equal to zero. The implied volatility is the Heston implied volatility defined by equation (9). The F-statistic tests the joint hypothesis of α = 0 and β = 1. The t-statistic in the last column is for the null hypothesis of β = 1. There is one observation for each Wednesday in the data period that follows an option expiration data, and the time interval covered by each observation extends from that Wednesday to the next option expiration date.

Realized Volatility	α	β	Adjusted R^2	F statistic	T statistic
Realized 1	0.036	0.6929	0.2862	29	−2.18
	(0.1363)	(4.925)			
Realized 2	0.021	0.5286	0.2953	83.99	−4.48
	(1.104)	(5.03)			
Realized 3	0.017	0.6306	0.3438	44	−3.28
	(0.82)	(5.6)			
Realized 4	−0.025	0.6564	0.3649	72.25	−3.26
	(−0.122)	(5.89)			

Source: Original Calculation

form, the Heston model may not necessarily generate better forecast than the simple Black-Scholes model.

If the implied volatility fails to predict future realized volatility, it may due to the reason that the realized volatility is totally unpredictable. For example, it is a random process; the past information contains no information in forecasting future volatility. To eliminate this possibility, researchers typically run a regression between the historical volatility and the realized volatility and see whether the realized volatility is predictable. The model is:

$$Vol_{realized}(t) = \alpha + \beta Vol_{historical}(t) + \varepsilon(t). \tag{11}$$

If the historical volatility contains information in forecasting future volatility, then β should be significantly different from zero. Moreover, if the historical volatility is an unbiased forecast of the realized volatility, then the joint hypothesis that $\alpha = 0$ and $\beta = 1$ cannot be rejected.

Table 4 reports our regression results. As we can see, the slope coefficient β is significantly different from zero for all measurements of volatility, indicating that the historical volatility does contain information in forecasting next period realized volatility. However, such a forecast is downward biased, all slope coefficients are significantly less than one. The largest slope coefficient is 0.586 for volatility measured by the Yang & Zhang estimator. The t-statistic for the null hypothesis that $\beta = 1$ is rejected at 99% confidence interval for all measurements of volatility. The joint hypothesis that $\alpha = 0$ and $\beta = 1$ is rejected at 99% confidence interval. Contrast to the results in Table 2 and Table 3, we find that historical volatility is a poor forecast of future realized volatility. The explanatory power of historical volatility is quite low comparing to the implied volatility in forecasting future realized volatility. When we run a

TABLE 4: FORECAST REGRESSION OF HISTORICAL VOLATILITY WITH DIFFERENT MEASUREMENT OF REALIZED VOLATILITY

$$Vol_{realized}(t) = \alpha + \beta Vol_{historical}(t) + \varepsilon(t)$$

OLS estimates with t-statistics for α = 0 and β = o (in parentheses). Different measurements of realized volatility are used. Realized 1 is computed from daily return, realized 2 is the standard deviation of Parkinson variance estimator, realized 3 is the standard deviation of Yang and Zhang variance estimator, realized 4 is the standard deviation of the sum-square of 5-minute intraday return. All volatilities are annualized. The realized volatility is computed under the restriction that the mean return is equal to zero. The historical volatility is computed the same way as the realized volatility. The historical volatility is computed from past one month S&P 500 return data up to the observation time. The F-statistic tests the joint hypothesis that α = 0 and β = 1. The t-statistic in the last column is for the null hypothesis that β = 1. There is one observation for each Wednesday in the data period that follows an option expiration data, and the time interval covered by each observation extends from that Wednesday to the next option expiration date.

Realized Volatility	α	β	Adjusted R^2	F statistic	T statistic
Realized 1	0.06	0.4748	0.2299	13.77	−4.735
	(3.387)	(4.28)			
Realized 2	0.062	0.2323	0.2155	374	−13.59
	(4.26)	(4.11)			
Realized 3	0.0558	0.586	0.2997	6.57	−3.59
	(3.48)	(5.087)			
Realized 4	0.0549	0.5365	0.2561	7.87	−3.95
	(3.734)	(4.578)			

Source: Original Calculation

TABLE 5: FORECAST REGRESSION OF BLACK-SCHOLES IMPLIED VOLATILITY AND HISTORICAL VOLATILITY WITH DIFFERENT MEASUREMENT OF REALIZED VOLATILITY

$$Vol_{realized}(t) = \alpha + \beta_1 Vol_{BS\_implied} + \beta_2 Vol_{historical}(t) + \varepsilon(t)$$

OLS estimates with t-statistics for $\alpha = 0$, $\beta_1 = 1$ and $\beta_2 = 0$ (in parentheses). Different measurements of realized volatility are used. Realized 1 is computed from daily return, realized 2 is the standard deviation of Parkinson variance estimator, realized 3 is the standard deviation of Yang and Zhang variance estimator, realized 4 is the standard deviation of the sum-square of 5-minute intraday return. All volatilities are annualized. The realized volatility is computed under the restriction that the mean return is equal to zero. The implied volatility is the Black-Scholes implied volatility from the closest ATM options. The historical volatility is computed in the same way as the realized volatility. The historical volatility is computed from past one month S&P 500 return data up to the observation time. The F-statistic tests the joint hypothesis that $\alpha = 0$, $\beta_1 = 1$ and $\beta_2 = 0$. There is one observation for each Wednesday in the data period that follows an option expiration data, and the time interval covered by each observation extends from that Wednesday to the next option expiration date.

Realized Volatility	α	β_1	β_2	Adjusted R^2	F statistic
Realized 1	0.01219	0.7826	0.0288	0.3522	16
	(−0.4586)	(−0.9523)	(0.1745)		
Realized 2	0.0084	0.6644	−0.03	0.3547	48
	(0.4272)	(−1.842)	(−0.345)		
Realized 3	0.0046	0.695	0.0412	0.414	24
	(0.2232)	(−1.527)	(0.218)		
Realized 4	−0.0148	0.7435	0.0081	0.4263	42
	(−0.7092)	(−1.459)	(0.05)		

Source: Original Calculation

TABLE 6: FORECAST REGRESSION OF HESTON IMPLIED VOLATILITY AND HISTORICAL VOLATILITY WITH DIFFERENT MEASUREMENT OF REALIZED VOLATILITY

$$Vol_{realized}(t) = \alpha + \beta_1 Vol_{Heston\_implied} + \beta_2 Vol_{historical}(t) + \varepsilon(t)$$

OLS estimates with t-statistics for $\alpha = 0$, $\beta_1 = 1$ and $\beta_2 = 0$ (in parentheses). Different measurements of realized volatility are used. Realized 1 is computed from daily return, realized 2 is the standard deviation of Parkinson variance estimator, realized 3 is the standard deviation of Yang and Zhang variance estimator, realized 4 is the standard deviation of the sum-square of 5-minute intraday return. All volatilities are annualized. The realized volatility is computed under the restriction that the mean return is equal to zero. The implied volatility is the Heston implied volatility defined by equation (9). The historical volatility is computed in the same way as the realized volatility. The historical volatility is computed from past one month S&P 500 return data up to the observation time. The F-statistic tests the joint hypothesis that $\alpha = 0$, $\beta_1 = 1$ and $\beta_2 = 0$. There is one observation for each Wednesday in the data period that follows an option expiration data, and the time interval covered by each observation extends from that Wednesday to the next option expiration date.

Realized Volatility	α	β_1	β_2	Adjusted R^2	F statistic
Realized 1	0.01	0.533	0.15	0.2828	121
	(0.37)	(2.28)	(0.85)		
Realized 2	0.023	0.4616	0.042	0.2854	174
	(1.17)	(2.56)	(0.45)		
Realized 3	0.022	0.4472	0.12	0.3468	196
	(1.02)	(2.26)	(1.12)		
Realized 4	0.093	0.5635	0.11	0.3589	155
	(0.043)	(3.18)	(0.68)		

Source: Original Calculation

regression between historical volatility and realized volatility, the adjusted R^2 decreases sharply, the highest R^2 is 0.2997 when realized volatility is measured by the Yang and Zhang estimator, the regression between implied volatility and realized volatility gives R^2 as 0.3629 for the similar measurement of realized volatility. When other measurement of realized volatility is used, the R^2 also decreases about 20% to 30% compared with the R^2 in the implied volatility regression. All results support the conclusion that historical volatility has less forecast ability than implied volatility in forecasting future realized volatility. The result is not surprising because option traders are generally institutional traders and may have better information in forecasting future volatility, so the implied volatility backed out from market option price is more closely correlated with future realized volatility.

It is also interesting to examine the relative forecast ability of implied volatility and historical volatility. Theoretically, if the two information sets both contain information in forecasting realized volatility, but one information set is the subset of another information set, then the regression between them and the realized volatility will lead to the slope coefficient of the first information set to be zero and the second to be one. The following regression is run:

$$Vol_{realized}(t) = \alpha + \beta_1 Vol_{implied}(t) + \beta_2 Vol_{historical}(t) + \varepsilon(t).$$

If $\beta_1 > \beta_2$, then implied volatility performs better than historical volatility in forecasting realized volatility, vise versa.

When both implied volatility and historical volatility are regressed to the realized volatility, the results are quite different from the univariate regression. Past volatility, in isolation, explains future realized volatility. However, once the implied volatility is added as an explanatory variable, the regression coefficient for past volatility β_2 drops dramatically from their values in the univariate regressions for all measurements of realized volatility. In fact, the t-statistic for the null hypothesis that $\beta_2 = 0$ cannot be rejected even at 90% confidence interval, indicating that historical volatility virtually contains no incremental information in forecasting future realized volatility in addition to the implied volatility. On the other hand, the regression coefficients for implied volatility β_1 does not change too much from their value in the univariate regressions. The t-statistic for the null hypothesis $\beta_1 = 0$ is rejected at 99% confidence interval when the Black-Scholes implied volatility is used. When the Heston implied volatility is used, the similar pattern can be found. The slope coefficients also drop 10% to 20% from their values in the univariate regressions, but are statistically different from zero at 99% confidence interval. The result shows that implied volatility dominates historical volatility in forecasting future realized volatility, which means that all information contained in past price has already been reflected in the option market. This can be regarded as evidence that option market process information efficiently. In other words, the implied volatility completely subsumes the information content of past volatility. Adding historical volatility as an explanatory variable does not increase the forecast ability of implied volatility. Actually the adjusted R^2 is even smaller than that in univiate regressions in terms of the relative performance of the Black-Scholes implied volatility and Heston implied volatility, the Black-Scholes implied volatility outperforms the Heston implied volatility in forecasting realized volatility. The regression R^2 is higher when Black-Scholes implied volatility is used for all measurements of realized volatility, meaning that the Black-Scholes implied volatility has more explanatory power in forecasting realized volatility. The F-statistic for the joint hypothesis that $\alpha = 0$, $\beta_1 = 1$ and $\beta_2 = 0$ is smaller for Black-Scholes implied volatility. The results is consistent with univarate test which shows that Black-Scholes implied volatility outperform Heston implied volatility in forecasting realized volatility.

6. Conclusion

This study examines how the measurement error in realized volatility and implied volatility affects the stability of the relation between implied volatility and realized volatility. We find implied volatility does contain information in forecasting realized volatility and the relation is stable under various measurements of realized volatility and implied volatility. In particular, the forecast ability can be improved by constructing a more accurate measurement of realized volatility, with the realized volatility constructed from intraday 5-minute returns being the most predictable. On the other hand, adding volatility as a random process does not increase the forecast ability of implied volatility. We find that the implied volatility computed from Black-Scholes model has higher explanatory power than that computed from Heston model. This is possibly because the Heston stochastic volatility model is too restrictive, and the actual true data generating process may deviate from Heston model significantly.

The historical volatility, on the other hand, has less explanatory power than the implied volatility in predicting realized volatility. Though the univariate regression of historical volatility to the realized volatility shows that historical volatility also has information in predicting realized volatility, the regression of the historical volatility and implied volatility simultaneously shows that the slope coefficient for the historical volatility is not statistically different from zero. This result shows that implied volatility dominates historical volatility in forecasting realized volatility, or that all the information contained in historical volatility has been reflected by the implied volatility, and the historical volatility has no incremental forecast ability. Our study shows that the option market process information efficiently.

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